Stability of Vortex Trails 98

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It is established that both the magnitude and the direction of the velocity of motion of infinite vortex systems (trails relative to fluid flow) which meets an obstacle producing trails depend on the mutual arrangement of both vortex chains. Actually, we will start from the same general formulas which gave us the complex velocity of each of the vortices of the two parallel vortex chains with equidistant vortices for both chains (distance $2l$) and with equal and opposite circulation Γ for each chain; namely, from

$$\left(\frac{dF}{dz}\right)_{z=z_0'} = \frac{\Gamma i}{4l} \cot \frac{\pi}{2l} (z_0' - z_0'') \qquad (1)$$

where z' , z'' represent the affixes of the vortices corresponding to the upper and lower chains and the index o relates to some initial vortex. Denoting by $2h$ the width of the street and putting $z_0'' = z_0' - 2d - 2ih$, we obtain the following three pairs of different components of the velocity of a moving trail+

$$U_S = \frac{\Gamma}{4l} \cot \varkappa\pi \ , \ V_S = 0; \quad U_Z = \frac{\Gamma}{4l} \tanh \varkappa\pi \ , \ V_Z = 0 \qquad (2)$$

$$U_A = \frac{\Gamma}{4l} \tanh \varkappa\pi \frac{1 + \tan^2 \lambda\pi}{\tan^2\lambda\pi + \tanh^2\varkappa\pi} \ ; \ V_A = -\frac{\Gamma}{4l} \tan\lambda\pi \frac{1 - \tanh^2 \varkappa\pi}{\tan^2\lambda\pi + \tanh^2 \varkappa\pi} \quad (3)$$

From formula (2) it is evident that as a consequence of $V_S = 0$, $V_Z = 0$, symmetrical and staggered trails move in the direction of the basic flow; asymmetric trails, in view of $V_A \neq 0$, move obliquely with respect to the basic flow. This does not altogether mean that the asymmetric system of vortices in both vortex chains changes the direction of its axis with respect to the direction of the basic streams but means only that such a trail moves parallel to itself, moving by degrees from the axis of symmetry of a streamlined body with which the axes of both the symmetric and staggered trails in all the time of their existence coincide.

If vortices of small displacement (of first or second but not fourth or much higher order) are added and the stability of the vortex system is studied then there is obtained, as is known, that the symmetric trail is determined unstable, the staggered may be stable if the Karman condition

$$\sinh \kappa\pi = 1, \quad \kappa = h/l \approx 0.281 \qquad (4)$$

is fulfilled,

and asymmetric trails if the more general condition [1]

$$\sinh \kappa\pi = \sin \lambda\pi$$

is fulfilled, which guarantees the existence of oblique flows of trails, is an occurrence which is not observed in nature and not stated for laboratory tests.

We have in view that theoretic investigation of the question of the motion and stability of vortex trails assumes, on the one hand, an ideal fluid and, on the other hand, considers infinite vortex trails not connected withaan obstacle which produces vortices in such time as these real causes produce observable staggered vortex trails which flow straight. But if it is taken into account that the water in which frequently is observed vortex trails is an almost ideal fluid and that vortex systems are studied far from the obstacle then all we believe we may give a purely theoretical explanation of the question: What produces the tendency of stable trails to appear exclusively in staggered but not in asymmetric order and to flow not slanting but straight?

Having in view that a very similar study by Kochin [2] in general excludes the stable state of staggered trails (but in one of the first of our reports [3] we showed that this relates to asymmetric trails) the statement of the above-mentionned question generally would be aimless if the result which gives us the theory is not in satisfactory accordance with experimental data. On one hand, these theoretical results now are used to construct a theory of vortex drag [4,5] which emanates from condition (4) guaranteeing the existence of "least unstable vortex formation". On the other hand, it follows not to forget that the mechanism of formation of the vortex system from a cylinder is such that the vortex trail must be, if not completely, at least almost symmetrical, then as we state that it is quickly made staggered and not disintegrated. We show that staggered arrangement is the last stage of an infinitely-many also-stable intermediate asymmetric trails. In other words, the latter, streaming obliquely, is stabilized and finally arranged in the form of staggered trails streaming straightly.

Considering formulas (2) and (3) earlier [1] , we established that for parameters λ and K which satisfy the stability conditions of staggered and asymmetric trails, therex is the property:

$\left|\vec{W}_{S}\right| > \left|\vec{W}_{A}\right| > \left|\vec{W}_{Z}\right|$ which shows that symmetric have more velocity, asymmetric trails have less velocity but staggered thails have smaller velocity. It is easy to deduce that the asymmetric stable trails are always narrower than stable staggered trails. These properties show that generalized asymmetric trails have, actually, a character intermediate between symmetric and staggered. All these facts also are close to observable test methods. That is whyywe proceed from such a displacement law which leads to stable staggered trails in unsymmetrical arrangement therefore to oblique flow system and we will seek the paths of separate vortices.

Oblique streaming vortex trails we already obtained for any "group" displacement (in each of two vortex chains is traced an infinite number of sections such that in every section is contained n vortices, the displacement of which in each succeeding section is repeated). For n = 2 (alternate displacement) we deduced the xa flow law (1) [3]. This effect appears in a more obvious way when n = 1 (identical displacements) in any case for the secondary component of velocity we find the formula [6]

$$V = \frac{\pi^2}{4l^2}\left\{\xi_0' - \xi_0''\right\}\cosh^{-2} x\pi$$

But in the considered particular case of group displacements for which we started from trails with staggered arrangement of vortices, the initial arrangement of the vortices was such that they always represented non-symmetric arrangements which produced, as we saw, $V \neq 0$. (On these asymmetric arrangements both Rosenhead [7] and Glauert [8] expressed the opinion that they may not be stable, by virtue of the fact that the velocity component perpendicular to the vortex chain changes the mutual vortex arrangement, i.e. destroys the trail, which does not appear the theoretical fact since the asymmetric trail, as we showed [1,5] may also be stable).

Thus, we will proceed from some staggered arrangement of the vortices and trace their path assuming as a law for the vortex displacement such identical displacements which, on the one hand, must move the vortex chains relative to one another but, on the other hand, expand (contract) to vortex trails. Our goal is to find the law which controls the trajectory of the vortices in their motion relative to the staggered arrangements which is not what is known as asymmetric arrangement.

In the same way, the assumed law for the displacement of vortices now has the form

$$z_0'' = z_0' - l \mp 2\xi - 2ih \pm 2i\eta \tag{6}$$

where ξ (the shift of the vortex chain), η (expansion or contraction correspondingly) of the trails already denote variable quantities. Then (6) may be written in the form

$$z_0'' = z_0' - 2l\{(\tfrac{1}{2} \pm \xi/l) + i(h/l \mp \eta/l)\} \tag{7}$$

or after substituting $\xi/l = \zeta$, $\eta'/l = \chi$, and also $\frac{1}{2} \pm \zeta = \varphi$ $\kappa \mp \chi = \psi$ in the form

$$z_0'' = z_0' - 2l(\varphi + i\psi) \tag{8}$$

Using formula (1), or what is the same, formula (3) ~~xhxx~~ which are valid for an asymmetric arrangement obtained from the symmetric for whhh it is necessary to replace the parameters λ, κ by φ, ψ we obtain the differential equations

$$\pm \frac{d\varphi}{dt} = \frac{\Gamma}{4l^2} \tanh\psi\pi \, \frac{1 + \tan^2\varphi\pi}{\tan^2\varphi\pi + \tanh^2\psi\pi}$$
$$\mp \frac{d\psi}{dt} = \frac{-\Gamma}{4l^2} \tan\varphi\pi \, \frac{1 - \tanh^2\psi\pi}{\tan^2\varphi\pi + \tanh^2\psi\pi} \qquad (9)$$

The trajectory, according to which the separate vortices move, we obtain after some transformation and elimination of dt from the equation

$$\frac{d\psi}{d\varphi} = \frac{\sin\varphi\pi\cos\varphi\pi}{\sinh\psi\pi\cosh\psi\pi} \qquad (10)$$

which has the integral

$$\sinh^2\psi\pi = \sin^2\varphi\pi + C \qquad (11)$$

To determine the constant of integration C we consider, on the one hand, that for $t = 0$, $\xi_o = 0$, $\eta_o = 0$, from which we obtain $\varphi_o = \frac{1}{2}$, $\psi_o = \chi$; on the other hand, since the initial arrangement was stable according to Karman we take into account condition (4). We then obtain $C = 0$.

Thus, we obtain the solution of (10) in the form

$$\sinh^2\psi\pi = \sin^2\lambda\pi \qquad (12)$$

from which, assuming $l = 1$, for the trajectory of the separate vortices we find the equation

$$\eta = h - \frac{1}{\pi}\ln\left|\sqrt{1 + \cos^2\xi\pi} \pm \cos\xi\pi\right| \qquad (13)$$

Since $\varphi = \frac{1}{2} + \xi/l$, then equation (12), naturally becomes

$$\sinh\psi\pi = \cos\xi\pi \qquad (14)$$

which coincides with the stability condition (5) for asymmetric trails in which condition it is necessary to put $\lambda = d/l + \frac{1}{2} = \frac{1}{2} + \mu$ from which we obtain, analogous to (14), that if we start from a staggered arrangement and it reduces to an asymmetric then the stability condition transforms to

$$\sinh \varkappa\pi = \cos \mu\pi$$

Thus we established that for simultaneous movement of vortex chains and contractions of trails the law which controls the trajectory of separate vortices is such that they describe stable trajectories. This means that all intermediate asymmetrical trails are stable which permit, without a break, trails to arrange their vortices in an ultimate staggered order.

With this stabilization, a vortex system passing through an infinite manifold of asymmetric configurations which alwarys are already staggered trails, decreases its velocity, and becoming possibly more broad, starts to move with the minimum velocity (2). This same broad trail corresponds to experimental fact that the observed staggered trail has a considerably larger width than the greatest transverse section of the streamline body. The same applies to the very slow flow of vortex trails far from the body. Near the body the trails undergo some oscillations both in respect to the mutual distance of the two vortex chains and in respect to the arrangement of the chains with respect to each other.

Stabilization of vortex trails is continued briefly; in that time the effect of oblique flow may be discovered only in the form of small displacements of the trail axis relative to the axis of s ymmetry of the body. As we showed, an a obstructing body also shows ists effect as a consequence ~~at of~~ af this displacement beyond the body is hardly noted.

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